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STRUCTURAL ANALYSIS AND OPTIMAL DESIGN OF A DYNAMIC ABSORBING BEAM

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The structural analysis and optimal design of a dynamic absorbing beam which is attached to the main beam with a viscoelastic layer or other mechanism of similar effect is presented. The dynamic stiffness matrix of a composite beam composed of two parallel beams with viscoelastic layer between them has already been derived and can be employed for the structural analysis. A simplified two-degree-of-freedom system is proposed for the optimal design of the dynamic absorbing beam. An example is included for demonstration and discussion.

1. INTRODUCTION

There are many ways to control the vibration of a beam structure such as a suspended mass with spring and dashpot as shown in Figure 1, a support isolation as shown in Figure 2 and a dynamic absorbing beam with viscoelastic layer as shown in Figure 3. The dynamic absorbing beam might have many advantages, some of which are (1) it is easy to build and maintain, (2) little space is required, (3) high performance can be achieved, (4) it is easy to remold to increase its performance and (5) there is no need to change the main beam structure. In view of these advantages, it is the purpose of this paper to present the structural analysis and the optimal design of a dynamic absorbing beam for engineering applications.

A dynamic absorbing beam system as shown in Figure 3 consists of a main beam, a dynamic absorbing beam, and a viscoelastic layer between them. Therefore the dynamic stiffness matrix of a layered beam with flexible core established by Chen and Sheu [1, 2] can be directly employed in the structural analysis of a dynamic absorbing beam system. Most of the vibrational level is dominated by the first two major modes. In general they might be any two of the first four lowest modes of a dynamic absorbing beam system. Therefore a simplified two-degree-of-freedom system may appropriately be used to describe the dynamic behavior of these two major modes of the dynamic absorbing beam system at the optimal condition. The properties of the simplified two-degree-of-freedom system and the important parameters for the optimal design of a dynamic absorbing beam system derived in this paper provide a practical design-guide. A free–free dynamic absorbing beam attached to a simply-supported main beam as an application example is presented for study and demonstration.

2. STRUCTURAL ANALYSIS

The dynamic stiffness matrix and the deflection functions of a composite beam element, which is composed of two parallel beams with a flexible viscoelastic layer in between,

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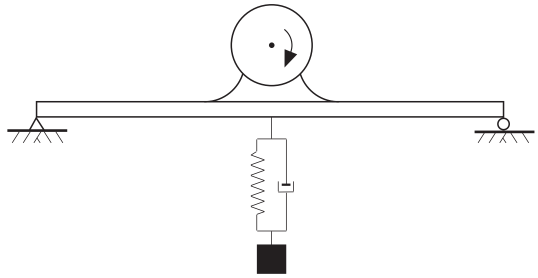

Figure 1. A suspended mass with spring and dashpot.

would be applied directly to the structural analysis of a dynamic absorbing beam system presented in this paper. The effects of the rotary inertia of mass, the shear distortion, the viscoelastic layer, the various damping components, and the boundary conditions at supports can all be included in the structural analysis. Applying the procedure of the direct stiffness method the assembly of the dynamic stiffness matrix of an entire structure can be accomplished by superposing the contributions from all the beam components affected by each individual nodal displacement. The nodal displacements can then be computed from the dynamic stiffness equations. The deflection at any position would be determined by the dynamic deflection functions accordingly. Unfortunately the explicit formula of the dynamic stiffness matrix and the dynamic deflection function of a composite beam element cannot be obtained due to their complexities. Only the implicit forms can be obtained by numerical computation for the practical application.

3. SIMPLIFIED TWO-DEGREE-OF-FREEDOM SYSTEM

Before going into the optimal analysis of a dynamic-absorbing beam, it is better to know the dynamic characteristics of a dynamic absorbing beam system such as the natural frequencies and the corresponding mode shapes, and the importance of each mode to the dynamic response. In most cases only the first two major modes, which might be any two of the first four lowest modes, should be taken into consideration in the optimal analysis of the dynamic absorbing beam system. The dynamic responses are significant at these two major natural frequencies and small at other natural frequencies due to the various damping components. If the mode shapes of these two major modes of the dynamic absorbing beam system can be assumed to be similar to each other as shown in Figure 4,

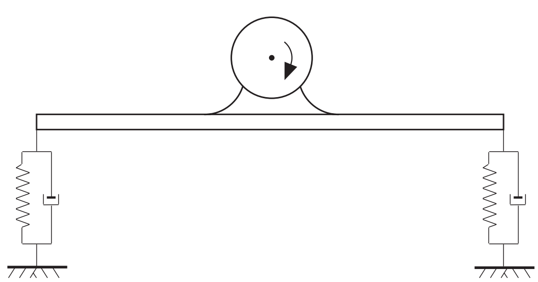

Figure 2. A support isolation system.

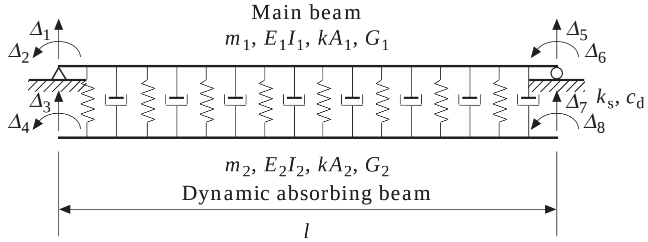

Figure 3. A dynamic absorbing beam system.

then the functions $\phi_1(x)$ and $\phi_2(x)$ can appropriately describe both the main beam and the dynamic-absorbing beam, respectively. Therefore the displacements of the main beam and the dynamic absorbing beam can be expressed in the following forms as

$$v_1(x, t) = \phi_1(x) y_1(t), \qquad v_2(x, t) = \phi_2(x) y_2(t), \tag{1}$$

where $\phi_1(x)$ and $\phi_2(x)$ represent the mode shapes and $y_1(t)$ and $y_2(t)$ represent the amplitudes for the main beam and the dynamic-absorbing beam, respectively.

If both the effects of the rotary inertia of the mass and the shear distortion of the beams are neglected, the kinematic and potential energies of the dynamic absorbing beam system can be given by

$$T = \tfrac{1}{2}\int_0^l m_1 \dot{v}_1^2 \,\mathrm{d}x + \tfrac{1}{2}\int_0^l m_2 \dot{v}_2^2 \,dx,$$

$$V = \tfrac{1}{2}\int_0^l E_1 I_1 v_1''^2 \,\mathrm{d}x + \tfrac{1}{2}\int_0^l E_2 I_2 v_2''^2 \,\mathrm{d}x + \tfrac{1}{2}\int_0^l k_s (v_1 - v_2)^2 \,\mathrm{d}x. \tag{2}$$

The definitions of the physical properties of the dynamic-absorbing-beam system represented by the symbols in Equation (2) and Figure 3 are given in Reference 1.

The virtual work done by the non-conservative forces including the external force and the damping force is given by

$$\delta W = \int_0^l p_1(x, t)\, \delta v_1 \,\mathrm{d}x + \int_0^l p_2(x, t)\, \delta v_2 \,\mathrm{d}x - \int_0^1 c_d (\dot{v}_1 - \dot{v}_2)\delta(v_1 - v_2) \,\mathrm{d}x, \tag{3}$$

where $p_1(x, t)$ and $p_2(x, t)$ represent the external forces acting on the main beam and the dynamic absorbing beam respectively.

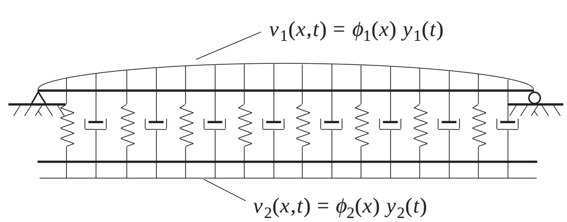

Figure 4. Displacement of dynamic absorbing beam system.

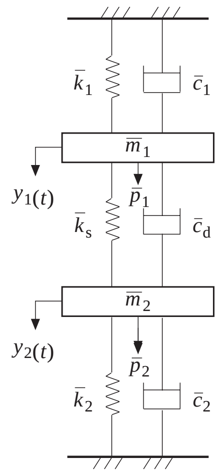

Figure 5. The simplified two-degree-of-freedom system.

Applying Hamilton's principle (or Lagrange's equations of motion) [3], the equations of motion of the dynamic absorbing beam system for these two major modes can be obtained and described by the generalized coordinates $y_1(t)$ and $y_2(t)$ as:

$$\begin{bmatrix} m_1^* & 0 \\ 0 & m_2^* \end{bmatrix}\begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} c_{d1}^* & -c_d^* \\ -c_d^* & c_{d2}^* \end{bmatrix}\begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix} + \begin{bmatrix} k_1^* + k_{s1}^* & -k_s^* \\ -k_s^* & k_2^* + k_{s2}^* \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} p_1^* \\ p_2^* \end{Bmatrix}, \tag{4}$$

where

$$m_i^* = m_i \int_0^l \phi_i^2(x)\,\mathrm{d}x, \qquad k_i^* = E_i I_i \int_0^l \phi_i''^2(x)\,\mathrm{d}x,$$

$$k_{si}^* = k_s \int_0^l \phi_i^2(x)\,\mathrm{d}x, \qquad c_{di}^* = c_d \int_0^l \phi_i^2(x)\,\mathrm{d}x, \tag{5}$$

$$k_s^* = k_s \int_0^l \phi_1(x)\phi_2(x)\,\mathrm{d}x, \qquad c_d^* = c_d \int_0^l \phi_1(x)\phi_2(x)\,\mathrm{d}x,$$

$$p_i^* = \int_0^l p_i(x,t)\phi_i(x)\,\mathrm{d}x, \qquad \text{all } i = 1, 2.$$

Therefore equation (4) shows that the dynamic behavior of these two major modes of a dynamic absorbing beam system would be described approximately by a simplified two-degree-of-freedom system as shown in Figure 5. The property constants denoted in Figure 5 are given by

$$\bar{m}_i = m_i^*, \qquad \bar{k}_s = k_s^*, \qquad \bar{c}_d = c_d^*, \qquad \bar{k}_i = k_i^* + k_{si}^* - k_s^*,$$

$$\bar{c}_i = \mathrm{c}_{di}^* - c_d^*, \qquad \bar{p}_i = p_i^*, \qquad \text{all } i = 1, 2. \tag{6}$$

If the external force is acting on the main beam only, equation (4) can be rewritten in the form:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{Bmatrix} + \begin{bmatrix} 2\xi\omega_2\, u & -2\xi\omega_2\, u\alpha\eta \\ -2\xi\omega_2\, \eta & 2\xi\omega_2 \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} \omega_1^2 + \omega_s^2\, u & -\omega_s^2\, u\alpha\eta \\ -\omega_s^2\, \xi & \omega_s^2 + \omega_2^2 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{Bmatrix} p_1^* / m_1^* \\ 0 \end{Bmatrix} \tag{7}$$

where

$$u = m_2 / m_1, \qquad \omega_i^2 = k_i^* / m_i^*, \quad i = 1, 2$$

$$\omega_s^2 = k_s / m_2, \qquad \alpha = \int_0^l \phi_2^2\, \mathrm{d}x \bigg/ \int_0^l \phi_1^2\, \mathrm{d}x$$

$$\eta = \int_0^l \phi_1\, \phi_2\, \mathrm{d}x \bigg/ \int_0^l \phi_2^2\, \mathrm{d}x \qquad \xi = c_d / (2m_2\, \omega_2) \tag{8}$$

Setting $\xi = 0$ and $p_1^* = 0$ for the case of free vibration, the natural frequencies and the corresponding mode shapes would be determined from equation (7) accordingly.

4. OPTIMAL DESIGN

If a unit harmonic force acts at the midspan of the main beam, the force and response vectors in equation (7) could be expressed in the following forms as

$$\{p^*\} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \mathrm{e}^{\mathrm{i}\omega t}, \qquad \{y\} = \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} \mathrm{e}^{\mathrm{i}\omega t}. \tag{9}$$

Substituting equation (9) into equation (7) will yield

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \end{Bmatrix} = \begin{Bmatrix} 1/m_1^* \\ 0 \end{Bmatrix}, \tag{10}$$

where

$$K_{11} = (-\omega^2 + \omega_1^2 + \omega_s^2\, u) + \mathrm{i}(2\xi\omega_2\, u\omega), \qquad K_{12} = (-\omega_s^2\, u\alpha\eta) - \mathrm{i}(2\xi\omega_2\, u\alpha\eta\omega),$$

$$K_{21} = (-\omega_s^2\, \eta) - \mathrm{i}(2\xi\omega_2\, \eta\omega), \qquad K_{22} = (-\omega^2 + \omega_2^2 + \omega_s^2) + \mathrm{i}(2\xi\omega_2\, \omega). \tag{11}$$

The dynamic magnification factors D_1 or D_2 is defined as the ratio of the displacement amplitude of the main beam or the dynamic absorbing beam to the static displacement at the midspan of the main beam and is given as

$$D_i = |\, Y_i \,| / \delta_{st}, \qquad i = 1, 2, \tag{12}$$

where δ_{st} represents the static displacement at the midspan of the main beam, which is equal to the value of Y_1 in equation (10) at zero frequency ($\omega = 0$) and given as

$$\delta_{st} = (\omega_s^2 + \omega_2^2) / \{m_1^* \, [(\omega_1^2 + \omega_s^2\, u)\, (\omega_s^2 + \omega_2^2) - \omega_s^4\, u\alpha\eta^2]\} \tag{13}$$

Finally the dynamic magnification factor D_1 can be determined and given by

$$D_1 = \{\delta_1^2\,(a^2 + b^2\xi^2)/\{\delta_2^2\,[(c + d\xi^2)^2 + e^2\xi^2]\}\}^{1/2} \tag{14}$$

where

$$\begin{gathered} a = -\beta^2 + g^2 + f^2, \qquad b = 2\beta f, \\ c = \beta^4 - [(1 + g^2) + (1 + u)f^2]\beta^2 + (g^2 + f^2 + uf^2g^2 + usf^4), \\ d = -4usf^2\beta^2, \qquad e = 2\beta f[-(1 + u)\beta^2 + (g^2 + 2sf^2)u + 1], \\ \delta_1 = (1 + uf^2)\,(g^2 + f^2) - u\alpha\eta^2 f^4, \qquad \delta_2 = g^2 + f^2, \\ s = 1 - \alpha\eta^2, \qquad g = \omega_2/\omega_1, \qquad f = \omega_s/\omega_1, \qquad \beta = \omega/\omega_1. \end{gathered} \tag{15}$$

There are two fixed points in the $D_1 - \beta$ curves described by equation (14) for a two-degree-of-freedom system having any value of the damping ratio ξ. The values of β at these two fixed points can be determined from the same value of D_1 given by equation (14) for $\xi = 0$ or $1{\cdot}0$ respectively i.e.,

$$a^2/c^2 = (a^2 + b^2)/[(c + d)^2 + e^2]. \tag{16}$$

Substituting equation (15) into equation (16) yields the following equation

$$D_8\,\beta^8 + D_6\,\beta^6 + D_4\,\beta^4 + D_2\,\beta^2 + D_0 = 0, \tag{17}$$

where

$$\begin{gathered} D_8 = u^2 + 2u - 2us, \\ D_6 = 4u^2s^2f^2 - 4u^2sf^2 - 2u^2g^2 - 2u^2f^2 + 2usf^2 + 6usg^2 - 6ug^2 - 2uf^2 + 2us - 2u, \\ D_1 = -6u^2s^2f^4 - 8u^2s^2f^2g^2 + 4u^2sf^2(f^2 + g^2) + 6u^2f^2g^2 + 6u^2g^4 \\ - 2usf^2 - 6usg^2 - 4usf^2g^2 - 6usg^4 + 6ug^2 + 6ug^4 + 2uf^2 + 4uf^2g^2, \\ D_2 = 4u^2s^2f^4g^2 + 4u^2s^2f^2g^4 - 2u^2sf^2g^4 - 4u^2sf^4g^2 - 2u^2f^2g^2 - 2u^2g^4 - 2u^2g^6 - 4u^2f^2g^4 \\ + 6usg^4 + 4usf^2g^2 + 2usf^2g^4 + 2usg^6 + 2usf^4g^2 - 2uf^2g^4 - 6ug^4 - 2ug^6, \\ D_0 = u^2s^2f^8 + 2u^2s^2f^4g^4 + 4u^2s^2f^6g^2 - 2u^2sf^6g^2 + u^2g^8 + 2u^2f^2g^6 - 2usg^6 \\ - 2usf^2g^4 + 2ug^6 + 2uf^2g^4. \end{gathered} \tag{18}$$

Equation (17) is of even order; therefore it has four-pair roots, and the real ones give two values of β. If these two fixed points are denoted by (D_a, β_a) and (D_b, β_b), the condition $D_a = D_b$ can yield the following equation

$$\begin{aligned} \beta_a^2\,\beta_b^2\,(\beta_a^2 - \beta_b^2)u^2 + [(-\beta_a^4 + \beta_b^4)\,(e_a + e_b) - (\beta_a^2 - \beta_b^2)\,(e_a^2\,s - e_a^2)]u \\ + (\beta_a^2 - \beta_b^2)\,(e_a + e_b)^2 = 0, \end{aligned} \tag{19}$$

where

$$e_a = k_s^*/k_1^*, \qquad e_b = (k_2^*/k_1^*)\left(\int_0^l \phi_1^2\,\mathrm{d}x \Big/ \int_0^l \phi_2^2\,\mathrm{d}x\right). \tag{20}$$

Equation (19) gives the relationship between the mass ratio u and the spring constant of the viscoelastic layer k_s, or the so-called constraint condition for the optimal design of a dynamic absorbing beam system. Two important optimal parameters u and k_s of the dynamic absorbing beam could be determined from the proposed two-degree-of-freedom system in Equation (19). The optimal damping ratio and the corresponding dynamic responses can then be calculated by the exact theory [1]. If $c_d = 0$, the optimal damping ratio η_s, which is the imaginary part of the complex stiffness coefficient k_s of the viscoelastic layer [1], could be determined by trial-and-error if the two maximum values of the $D_1 - \beta$ curve for the first two major modes are just at or very close to these two fixed points [4].

5. APPLICATION AND DISCUSSION

5.1. APPLICATION: A FREE–FREE DYNAMIC ABSORBING BEAM

A free–free dynamic absorbing beam is attached to the simply-supported main beam with the viscoelastic layer as shown in Figure 3. The properties of this dynamic-absorbing beam system are given as: (a) Main beam (square section): Young's modulus $E = 2 \times 10^{11}$ N/m^2, Poisson ratio $v = 0{\cdot}3$, beam length $l = 1$ m, beam width $b = 1$ cm, beam depth $d = 1$ cm, shape factor for shear $k' = 0{\cdot}87$, mass per unit length $m = 0{\cdot}76$ kg/m. (b) Dynamic absorbing beam (rectangular section): $E = 2 \times 10^{11}$ N/m^2, $v = 0{\cdot}3$, $l = 1$ m, $b = 1$ cm, $k' = 0{\cdot}87$. Ten cases are under investigation, these being $d = 0{\cdot}072$, 0·175, 0·295, 0·440, 0·448, 0·510, 0·614, 0·650, 0·900 and 1·100 cm. All the masses of these cases are proportional to the main beam by the beam depth d. (c) Viscoelastic layer: Let $c_d = 0$, k_s and η_s are as requested for optimal design to achieve the minimum vibrational level at the midspan of the main beam where the excitation is applied.

The constraint condition defined by Equation (19) gives the relationship between the parameters u and k_s for the optimal design as shown in Figure 6 obtained by numerical computation. The optimal value of η_s can then be determined by trial-and-error and the result are also shown in Figure 6. The corresponding dynamic magnification factors D_1 and D_2 are also calculated and shown in Figures 7 and 8 respectively. The natural frequencies for these ten optimal cases are calculated by both the exact theory [1] and the approximate method presented in this paper, and the results are given in Table 1. The values in the brackets in Table 1 are calculated by the latter method and indicate the two

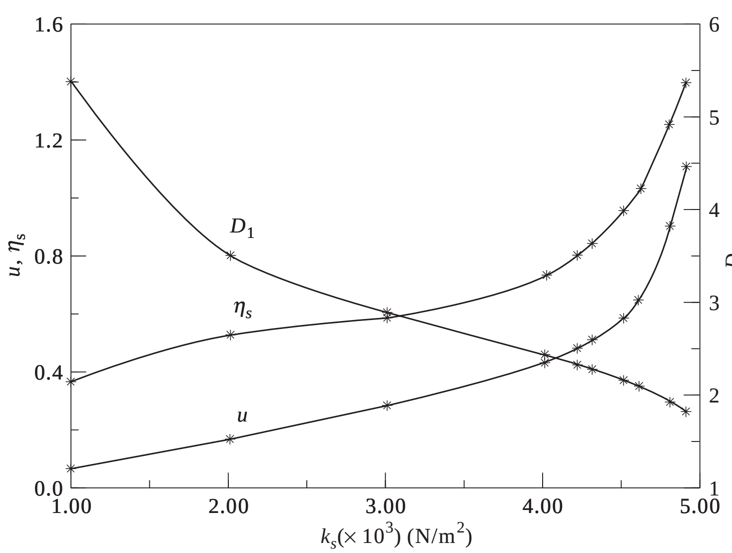

Figure 6. Curves of D_1, u and η_s versus k_s.

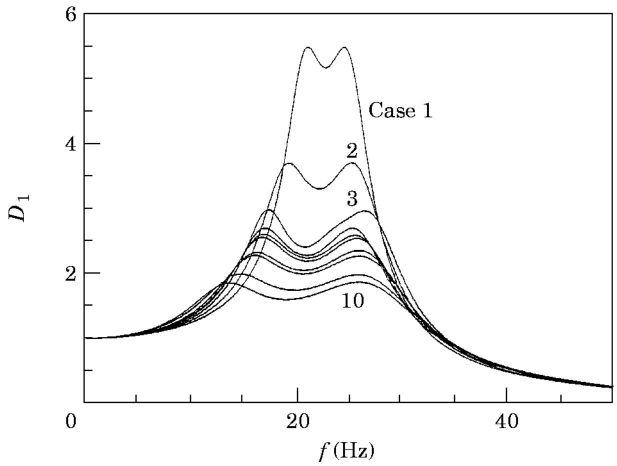

Figure 7. Dynamic magnification factor of main beam.

major modes. Both results show excellent agreement particularly for the cases of the harder layer (or higher value of k_s). The exact mode shapes for cases 3 and 8 are shown in Figure 9. The overall (or effective) damping ratios are also calculated by the exact theory and the results are given in Table 2. The displacement functions $\phi_1(x)$ and $\phi_2(x)$ are assumed as a sine function and a horizontal line for the main beam and the dynamic absorbing beam, respectively, for approximation. Sometimes $\phi_1(x)$ and $\phi_2(x)$ of the first mode calculated by the exact theory could give a better result, particularly for a softer layer.

The vibration of the dynamic absorbing beam defined by D_2 and shown in Figure 8 is not very significant, which might be an another advantage of the dynamic absorbing beam in practice. It can be seen in Figure 6 that the dynamic magnification factor D_1 is almost inversely proportional to the design parameters k_s, η_s, and u. Both u and η_s will increase tremendously with k_s if $k_s \geqslant 4 \times 10^3$ N/m^2; therefore it is not practical to control the vibration or to achieve a smaller value of D_1 for this situation. The optimal condition will

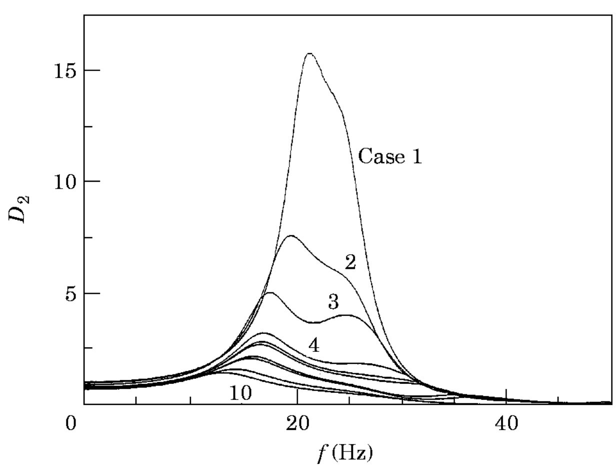

Figure 8. Dynamic magnification factor of dynamic absorbing beam.

TABLE 1

Natural frequencies (Hz) of the dynamic absorbing beam system at optimal condition

Case	d (cm)	u	k_s (N/m^2)	η_s	First mode	Second mode	Third mode	Fourth mode
1	0·072	0·072	$1{\cdot}0 \times 10^3$	0·370	19·59 (19·77)	23·91	25·60 (25·30)	29·73
2	0·175	0·175	$2{\cdot}0 \times 10^3$	0·480	17·47 (17·56)	19·46	21·34	26·34 (26·01)
3	0·295	0·295	$3{\cdot}0 \times 10^3$	0·540	16·08 (16·15)	18·14	23·51	27·30 (26·62)
4	0·440	0·440	$4{\cdot}0 \times 10^3$	0·734	15·08 (15·13)	17·32	26·48 (27·24)	29·81
5	0·488	0·488	$4{\cdot}2 \times 10^3$	0·801	14·57 (14·71)	16·85	26·87 (27·29)	31·28
6	0·510	0·510	$4{\cdot}3 \times 10^3$	0·843	14·51 (14·55)	16·58	27·00 (27·33)	32·08
7	0·614	0·614	$4{\cdot}5 \times 10^3$	0·982	13·59 (13·61)	15·54	27·17 (27·25)	36·13
8	0·650	0·650	$4{\cdot}6 \times 10^3$	1·020	13·35 (13·38)	15·27	27·24 (27·28)	37·68
9	0·900	0·900	$4{\cdot}8 \times 10^3$	1·270	11·69 (11·71)	13·26	27·18 (27·16)	49·31
10	1·100	1·100	$4{\cdot}9 \times 10^3$	1·390	10·72 (10·73)	12·11	27·13 (27·12)	59·26

be spoiled if $k > 4{\cdot}9$ N/m^2. It is the reason that both the main beam and the dynamic absorbing beam will move at the same direction due to the hard layer (high value of k_s) tying them together while vibrating, and thus the energy absorbed by the viscoelastic layer is very limited. All of these discussions could provide a rule of thumb for the optimal design of a dynamic absorbing beam system.

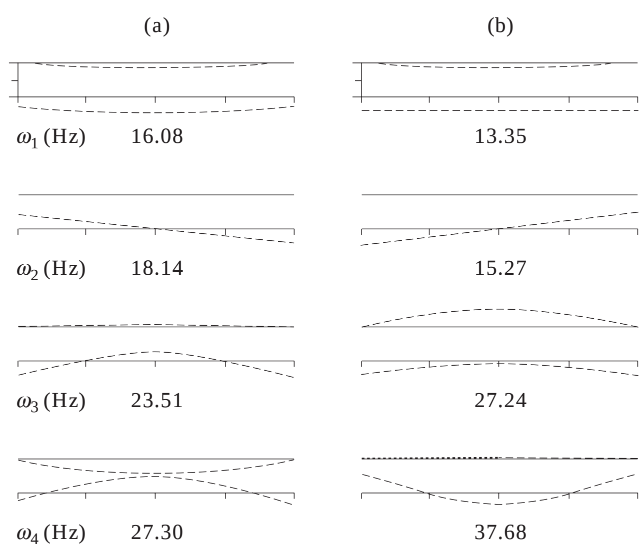

Figure 9. Mode shapes for (a) $d = 0{\cdot}295$ and (b) $d = 0{\cdot}650$ cm.

TABLE 2

Over-all (or effective) damping ratio of the dynamic absorbing beam system at optimal condition

Case	First mode	Second mode	Third mode	Fourth mode
1	0·1057	0·0921	0·0842	0·0075
2	0·1275	0·1175	0·1909	0·0191
3	0·1430	0·1230	0·1402	0·0258
4	0·1742	0·1302	0·1454	0·0246
5	0·1870	0·1414	0·1342	0·0209
6	0·1925	0·1506	0·1362	0·0197
7	0·2197	0·1731	0·0993	0·0116
8	0·2251	0·1803	0·0906	0·0541
9	0·2654	0·2084	0·0478	0·0116
10	0·2841	0·2171	0·0299	0·0130

6. CONCLUSIONS

Some important conclusions can be drawn from this study and are given as:

(1) The dynamic absorbing beam which is attached to the main beam with a viscoelastic layer between them can reduce the vibration of the main beam very effectively, since most of the dynamic magnification factors of the main beam can decrease tremendously and are, in general, less than ten or are of even smaller values (refer to Figure 7).

(2) The dynamic magnification factor of the dynamic absorbing beam is limited (refer to Figure 8) compared with other types of absorbers [5]. Therefore the whole system including both the main beam and the dynamic absorbing beam would remain in a somewhat quiet condition vibration.

(3) In view of the two previous conclusions and the advantages mentioned in the Introduction, the dynamic absorbing beam might have a great potential in engineering application.

(4) The exact theory of the layerd beam with flexible core [1] and the approximate model of the simplified two-degree-of-freedom system presented in this paper would provide an efficient tool for the structural analysis and the optimal design of a dynamic absorbing beam system in practice.

(5) The natural frequencies and the mode shapes of a dynamic absorbing beam system can be determined exactly in advance by the theory of the layer beam with flexible core. The approximate natural frequencies of the first two major modes could also be found from the model of the simplified two-degree-of-freedom system.

(6) Most of the vibrational level of the main beam is dominated by the first two major modes. The first major mode can be used as the displacement functions ϕ_1 and ϕ_2 given in Equation (1) for the optimal design by using the simplified two-degree-of-freedom system. These displacement functions could also be assumed by two known functions similar to the first major mode for approximation, such as the sine function and a horizontal straight line as used in the example.

(7) The mass ratio (u) of the dynamic absorbing beam should be assumed at first, the stiffness (k_s) of the visocelastic layer can then be determined by the constraint equation (refer to Equation (19)) based on the simplified two-degree-of-freedom system. Finally the optimal damping ratio (η_s), which is the imaginary part of the complex stiffness (k_s), would be calculated by the theory of the layered beam using a trial-and-error scheme.

(8) When the optimal parameters u, k_s, and η_s are all found, the dynamic responses (or the dynamic magnification factors D_1 and D_2) can be calculated easily from the theory of the layered beam.

(9) The optimal design would be spoiled if the value of the mass ratio (u) is greater than a certain limit (refer to Figure 6). Fortunately it is not a problem, since the mass ratio should not be much greater than 0·2 in practice.

(10) The design and manufacture of the dynamic absorbing beam, particularly the viscoelastic layer, will be considered as a research topic for future study. Of course more experimental work should be encouraged for this purpose.

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